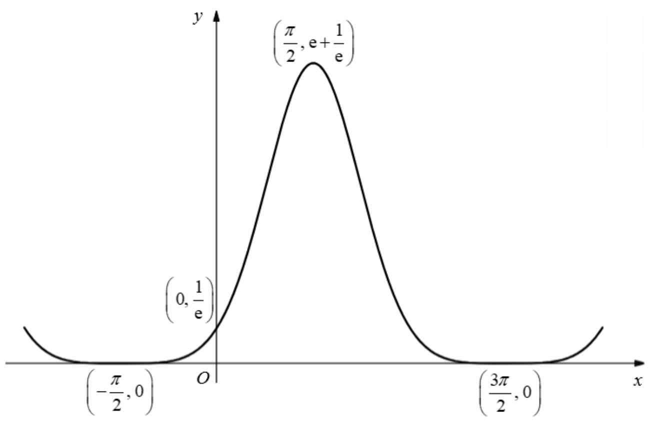
9811 Introduction to AEA worksheet

Mark scheme

Question Number	Scheme		Marks	AOs
1(a)	$\{k=\}\dfrac{7}{2}$	cao but condone, e.g., $x \neq \dfrac{7}{2}$	B1	2
			(1)	
(b)	$\dfrac{10}{2x-7} \to \ldots\ln(2x-7)$ or $\to \ldots\ln(7-2x)$	Attempts to integrate the reciprocal term $\dfrac{10}{2x-7} \to \ldots\ln(2x-7)$ or $\dfrac{10}{7-2x} \to \ldots\ln(7-2x)$	M1	1
	$\displaystyle\int\left(\frac{10}{2x-7}+3\right)\mathrm{d}x = 5\ln(2x-7)+3x \quad \{+c\}$	Correct integration ignoring the treatment of their limits, so allow for $5\ln(2x-7)+3x$ or, e.g., $\displaystyle\int\left(\frac{10}{7-2x}+3\right)\mathrm{d}x \to -\frac{10}{2}\ln(7-2x)+3x$ Ignore any inclusion of $+c$	A1	1
	$\displaystyle\int_{8}^{11.5}\left(\frac{10}{2x-7}+3\right)\mathrm{d}x = \left[\ldots\ln(2x-7)+3x\right]_{8}^{11.5}$ $=(\ldots\ln(2(11.5)-7)+3(11.5))$ $-(\ldots\ln(2(8)-7)+3(8))$ $=\ldots$ **or** $\displaystyle\int_{-4.5}^{-1}\left(\frac{10}{7-2x}+3\right)\mathrm{d}x = \left[\ldots\ln(7-2x)+3x\right]_{-4.5}^{-1}$ $=(\ldots\ln(7-2(-1))+3(-1))$ $-(\ldots\ln(7-2(-4.5))+3(-4.5))$ $=\ldots$	Complete strategy to find the required area.	M1	3
	$=5\ln\dfrac{16}{9}+\dfrac{21}{2}$	cso but allow exact equivalents e.g. $=10\ln\dfrac{4}{3}+\dfrac{21}{2}$	A1	2
			(4)	
			(Total 5 marks)	

Question Number	Scheme		Marks	AOs
2(a)	$y = \sin x \, \mathrm{e}^{\sin x} + a$	Correct function for y in terms of x which may be implied by correct a or b	**M1**	1
	$a = \frac{1}{\mathrm{e}}$ or $b = \mathrm{e} + \frac{1}{\mathrm{e}}$ o.e.	Either value correct	**A1**	3
	$a = \frac{1}{\mathrm{e}}$, $b = \mathrm{e} + \frac{1}{\mathrm{e}}$ o.e.	Both values correct	**A1**	2
			(3)	
(b)		Shape – must be only in quadrants 1 and 2 with two of the four intercepts/stationary points labelled correctly following through on their a and b	**B1ft**	3
		Fully correct, with all intercepts and turning points labelled and roughly symmetric about $x = \frac{\pi}{2}$	**B1**	2
			(2)	
			(Total 5 marks)	

Question Number	Scheme		Marks	AOs
3(a)	$\dfrac{(-4)(-5)(-6)\ldots(-4-r+1)(-1)^r}{r!}$	Correct form for the coefficient (or term) condoning absence of $(-1)^r$	B1	1
	$=\dfrac{4.5.6\ldots(r+3)}{1.2.3\ldots r}$	Absorbs the $(-1)^r$ across the terms in the numerator	M1	3
	$=\dfrac{\cancel{4}.\cancel{5}.\cancel{6}\ldots\cancel{r}\,(r+1)(r+2)(r+3)}{1.2.3\ldots\cancel{r}}$ $=\dfrac{(r+1)(r+2)(r+3)}{6}$ *	cso Convincing proof with evidence of the correct terms in the numerator and some clear cancelling	A1*	2
			(3)	
(b)	$\dfrac{3(r+1)(r+2)(r+3)}{6}$ $+\dfrac{2r(r+1)(r+2)}{6}$ $-\dfrac{5(r-1)r(r+1)}{6}$	Two of the three required terms correct	B1	1
	$=\dfrac{(r+1)\left[3(r+2)(r+3)+2r(r+2)-5r(r-1)\right]}{6}$ $=\dfrac{(r+1)\left[3r^2+15r+18+2r^2+4r-5r^2+5r\right]}{6}$	Expands and collects like terms in the numerator to achieve product of a linear and a quadratic factor where the quadratic term may or may not cancel	M1	3
	$=\dfrac{(r+1)(24r+18)}{6}=(r+1)(4r+3)$ *	cso	A1*	2
			(3)	

Question Number	Scheme		Marks	AOs
(c)	e.g. $\left(\sum_{r=0}^{\infty}(r+1)(4r+3)=\right)(3+)\,14+33+60+...$ or $\left(\sum_{r=0}^{\infty}(r+1)(4r+3)x^r=\right)(3+)\,14x+33x^2+60x^3+...$	Attempts to use the given expression for S and their answer to part (b) to relate the series to a sum – the LHS and the 3 might not be seen or they might have simply deduce the $\left(-\frac{1}{2}\right)$	**M1**	3
	$3-7+\frac{33}{4}-\frac{15}{2}+\frac{95}{16}-...=\sum_{r=0}^{\infty}(r+1)(4r+3)\left(-\frac{1}{2}\right)^r$ $=\frac{3+2\left(-\frac{1}{2}\right)-5\left(-\frac{1}{2}\right)^2}{\left(1-\left(-\frac{1}{2}\right)\right)^4}$	Deduces the correct sum and attempts to link to $\frac{3+2x-5x^2}{(1-x)^4}$	**dM1**	3
	$=\frac{3/4}{\left(3/2\right)^4}=\frac{4}{27}$	cao	**A1**	2
			(3)	
(d)	Either: • Requires substitution of $x=-2$ but the expansion is only valid for $\lvert x\rvert<1$ • The series diverges	Any correct reason.	**B1**	2
			(1)	
			(Total 10 marks)	

Question Number	Scheme		Marks	AOs
4(a)	$\left(x-a_1\right)^2+\left(y-b_1\right)^2=2^2$	Correct form for equation with correct radius.	**M1**	1
	$\left(x-2\right)^2+\left(y-2\right)^2=4$	Correct answer only.	**A1**	1
			(2)	
(b)	$\left(a_n-a_{n-1}\right)^2+\left(b_n-b_{n-1}\right)^2=\left(r_n+r_{n-1}\right)^2$	Attempts Pythagoras using the difference in centres and sum of radii. May be implied.	**M1**	3
	$\left(a_n-a_{n-1}\right)^2+\left(b_n-b_{n-1}\right)^2=\left(b_n+b_{n-1}\right)^2$ $\left(a_n-a_{n-1}\right)^2+b_n^2-2b_nb_{n-1}+b_{n-1}^2=b_n^2+2b_nb_{n-1}+b_{n-1}^2$ $\left(a_n-a_{n-1}\right)^2=4b_nb_{n-1}$	Expands and collects like terms. Condone if they also expand the a_n bracket.	**dM1**	3
	$a_n-a_{n-1}=2\sqrt{b_nb_{n-1}}$ *	cso Penalise inclusion of $\pm$	**A1***	2
			(3)	
(c)	$a_n-a_{n-1}=2\sqrt{2^n2^{n-1}}=\ldots$	Substitutes in for b_n and b_{n-1} and attempts to simplify.	**M1**	1
	$a_n=a_{n-1}+2^n\sqrt{2}$	cso – no need to state $\mathrm{f}(n)$ but requires correct work.	**A1**	2
			(2)	
(d)	$m=\dfrac{b_n-b_{n-1}}{a_n-a_{n-1}}=\left(\dfrac{2^n-2^{n-1}}{"2^n"\sqrt{2}}=\right)\dfrac{2^{n-1}}{"2^n"\sqrt{2}}$	Attempts the gradient between successive centres. Must be generalised. Follow through on their $\mathrm{f}(n)$	**M1**	2
	$m=\dfrac{1}{2\sqrt{2}}$	Correct gradient independent of n.	**A1**	3
	(Each line segment joining successive centres has a shared point and) The gradient between successive centres is constant. Hence the centres lie on a straight line.	Fully correct work with justification that the centres lie on a straight line.	**A1** **(S+)**	2
			(3)	

Question Number	Scheme		Marks	AOs
(e)	$y-2="\frac{1}{2\sqrt{2}}"(x-2)$	Correct equation of the straight line through centres for their gradient.	B1	3
	$2^n-2="\frac{1}{2\sqrt{2}}"(a_n-2)$	Substitutes $x=a_n$ and $y=2^n$ into their straight line.	M1	3
	$a_n=2+2\sqrt{2}(2^n)-4\sqrt{2}$ $a_n=2+4\sqrt{2}(2^{n-1}-1)$ *	Achieves the given answer with no errors and at least one intermediate step.	A1*	2
			(3)	
	Award S1 for: • a fully correct solution that is succinct but does not include the S+ point • a fully correct solution that may be laboured but includes the S+ point • a succinct solution that scores 11+ marks that includes the S+ point.		S1	2
S+ for good proof of linearity of the centres.				
			(Total 13+1 marks)	
Alt (e) by sum of geometric series	$a_n=2+\sqrt{2}\left(\sum_{i=2}^{n}"2^i"\right)$ for $n\geqslant 2$	Correct equation of the straight line through centres for their $\mathrm{f}(n)$	M1	3
(There are other similar approaches to this that should be marked in the same way)	$\sum_{i=2}^{n}2^i=\sum_{i=0}^{n}2^i-\sum_{i=0}^{1}2^i=\frac{1(2^{n+1}-1)}{2-1}-\frac{1(2^2-1)}{2-1}$ $\{=2^{n+1}-4\}$	Attempts sum of a geometric series for their $\mathrm{f}(n)$	dM1	3
	$a_n=2+\sqrt{2}(2^{n+1}-4)$ $a_n=2+4\sqrt{2}(2^{n-1}-1)$ *	Achieves the given answer with no errors and at least one intermediate step.	A1*	2
			(3)	

Question Number	Scheme		Marks	AOs
5(a)	$x = Ut\cos\alpha$ $y = \{H+\}Ut\sin\alpha - \frac{g}{2}t^2$	cao Condone missing $H+$	B1 B1	1 1
	$t = \frac{x}{U\cos\alpha}$ $\Rightarrow y = \{H+\}x\frac{\sin\alpha}{\cos\alpha} - \frac{g}{2}\left(\frac{x}{U\cos\alpha}\right)^2$	Full attempt to eliminate t	M1	3
	$\Rightarrow y = \{H+\}x\tan\alpha - \frac{gx^2}{2U^2}\sec^2\alpha$ **and** $\sec^2\alpha = 1+\tan^2\alpha$ used $\Rightarrow y = \{H+\}x\tan\alpha - \frac{gx^2}{2U^2}\left(1+\tan^2\alpha\right)$	$\sec^2\alpha$ seen before replacing with $1+\tan^2\alpha$ and $\frac{\sin\alpha}{\cos\alpha}$ replaced with $\tan\alpha$	dM1	1
	$\Rightarrow y = H + x\tan\alpha - \frac{gx^2}{2U^2}\left(1+\tan^2\alpha\right)$ *	cso The $H+$ may be recovered if justified.	A1*	2
			(5)	
(b)	$0 = H + x\tan\alpha - \frac{gx^2}{2U^2}\left(1+\tan^2\alpha\right)$ $\Rightarrow 0 = \frac{dx}{d\alpha}\tan\alpha + x\sec^2\alpha - \frac{\cancel{2}gx}{\cancel{2}U^2}\frac{dx}{d\alpha}\left(1+\tan^2\alpha\right)$ $-\frac{gx^2}{\cancel{2}U^2}\left(\cancel{2}\tan\alpha\sec^2\alpha\right)$	Sets $y=0$ and attempts to differentiate implicitly wrt α Correct differentiation	M1 A1	3 1
	$\Rightarrow 0 = x\sec^2\alpha - \frac{gx^2}{U^2}\left(\tan\alpha\sec^2\alpha\right)$ $\left(\Rightarrow 0 = R\sec^2\beta - \frac{gR^2}{U^2}\left(\tan\beta\sec^2\beta\right)\right)$	Sets $\frac{dx}{d\alpha}=0$ (may occur at the same time as differentiating in which case the A1 may also be scored)	dM1 (S+)	3
	$\Rightarrow 0 = R\sec^2\beta\left(1-\frac{gR}{U^2}\tan\beta\right)$ $\Rightarrow R = \ldots$	Attempts to factorise and make R the subject or rearranges to make R the subject with correct algebra (may still be in terms of x and α)	ddM1	1
	$\Rightarrow R = \frac{U^2}{g\tan\beta} = \frac{U^2\cot\beta}{g}$ *	cso may change to R and β at the final stage	A1*	2
			(5)	

Question Number	Scheme		Marks	AOs
(c)	$0 = H + \frac{U^2 \cot\beta}{g}\tan\beta - \frac{g\left(\frac{U^2\cot\beta}{g}\right)^2}{2U^2}\left(1+\tan^2\beta\right)$	Replaces x with R and y with 0 and $\tan\alpha$ with $\tan\beta$ in the given answer to (a) but condone use of α throughout	**M1**	3
	$\frac{U^2\cot\beta}{g}\tan\beta \to \frac{U^2}{g}$ **and** e.g. $\frac{g\left(\frac{U^2\cot\beta}{g}\right)^2}{2U^2}\left(1+\tan^2\beta\right) \to \frac{U^2\text{cosec}^2\beta}{2g}$	Cancels $\tan\beta$ and $\cot\beta$ (may not be seen due to trivial cancellation) and simplifies $\cot^2\beta\left(1+\tan^2\beta\right)$ to $\text{cosec}^2\beta$ o.e.	**dM1**	3
	e.g. $\text{cosec}^2\beta = \frac{2gH+2U^2}{U^2}$ o.e.	Correct intermediate equation in one trig term	**A1 (S+)**	1
	$\Rightarrow 1+\cot^2\beta = \frac{2gH}{U^2}+2$ $\Rightarrow \cot^2\beta = \frac{2gH}{U^2}+1$	Makes $\cot^2\beta$, $\cot\beta$, $\tan\beta$ or $\tan^2\beta$ the subject	**ddM1**	3
	$\Rightarrow \tan^2\beta = \frac{U^2}{2gH+U^2}$ $\Rightarrow \tan\beta = \sqrt{\frac{U^2}{2gH+U^2}}$ $\Rightarrow \beta = \arctan\left(\frac{U}{\sqrt{2gH+U^2}}\right)$*	cso with sufficient steps shown proceeding to the given answer with no errors.	**A1***	2
			(5)	
	Award S1 for: • a fully correct solution that is succinct but does not include any S+ points • a fully correct solution that may be laboured but includes an S+ point • a succinct solution that scores 10+ marks that includes an S+ point.		**S1**	2

S+ In part (b), changes to R and β when $\frac{dx}{d\alpha} = 0$ is used.

S+ In part (c) clear and concise cancelling of terms and/or use of trigonometric identities.

(Total 15+1 marks)